MATHEMATICS AS ONE OF THE HUMANITIES

by William Carey, Ad Fontes Academy

With some regularity parents come up to me and let me know that they hated math in high school. Such are the occupational hazards of teaching math. I console them, though, by suggesting that the thing they hated in high school probably wasn't math, and that if they took a stab at actual math, they might find that they like it.

The cultural practices of adults gathering to make music or discuss books together seem normal. The cultural practice of gathering to do math together for pleasure and nourishment seems, well, odd. Why is that? Inasmuch as it's *odd*, it's odd because of how we were taught to think about mathematics.

When we teach a subject, we do at least two things: we teach the subject, and we teach our students what it means to be a student of the subject. So, when a history teacher has the students read Herodotus, she both gives them a picture of the Persian wars, and also a vision of what it means to read and write history. By selecting Herodotus and not selecting, say, a long list of facts and figures about the chronology of the Persian wars, the history teacher is making claims about what it means to be a historian. Similarly, when a literature teacher chooses to give his students an essay exam instead of having them answer multiple choice questions, he is making a definite claim about what it means to be a reader and engage in the cultural practice of thoughtful reading. When the math teacher assigns problems one through thirty, just the odds, for homework, he is making a definite—though perhaps unconscious—claim about what it means to be a mathematician.

So, what does the shape of a modern math class teach students culturally about being a mathematician? Here I'll lean on Paul Lockhart's most excellent essay "A Mathematician's Lament." Lockhart describes an imaginary music class like this:

Music class is where we take out our staff paper, our teacher puts some notes on the board, and we copy them or transpose them into a different key. We have to make sure to get the clefs and key signatures right, and our teacher is very picky about making sure we fill in our quarter-notes completely. One time we had a chromatic scale problem and I did it right, but the teacher gave me no credit because I had the stems pointing the wrong way.

Bill Carey has been involved in the life of Ad Fontes Academy since he was in high school. After five years of teaching, Mr. Carey took a sabbatical writing computer programs for a defense contractor to better understand how adults think about and use mathematics. Drawn back to the classroom, Bill is in his fourth year of his second tour at Ad Fontes and second year as dean of instruction.

Music teachers might well shudder in horror. His contention—right I think—is that we teach math in an analogous way. Modern mathematical education teaches students that mathematics is unreal, authoritarian, and Sisyphean.

MATH IS UNREAL

A chapter in our old Algebra II book opened by explaining that linear inequalities were relevant because you could use them in your job . . . as a bowling pin manufacturing quality control inspector. If I'm a student, I'm out. I don't want to be a bowling pin manufacturing quality control inspector, so I guess I don't need to learn this! And how many of those are there in the world? Five? Ten? Out of billions of people.

There are examples like this throughout math textbooks. The fundamental difficulty with these imagined problems is that the context is unreal. If you want to . . . calculate the height of a jumping armadillo, you should learn quadratic equations with negative coefficients. Well, I don't, so I guess I shouldn't. Instead of genuine problems faced by adults, these problems are ruses to trick students into practicing what math teachers want them to practice. What does that teach students about how mathematicians think about *truth*?

We are purveyors of hypothetical imperatives that patronize and embarrass students. The whole thing is a sort of kabuki theatre, in which math teachers pretend (I hope?) to think these problems meaningful, and students pretend not to laugh at them. Not, perhaps, what we're aiming for.

MATH IS AUTHORITARIAN

"They" have invaded our math classrooms. It's fascinating how students want to know what the illusory "they"—by which they mean the authors of their textbook—want the students to do. Think for a minute about what that says about mathematical community. The most important people in the community are a faceless —and often nameless—committee, who bestow their judgements impersonally, and often without explanation. The aim of math is not to convince one's self, or one's peers, or even one's teacher, but to placate some ethereal "they."

Imagine, for a minute, a literature textbook that featured, after a chapter of *Pride and Prejudice* a series of questions like: "Do Wickham and Lydia have a good marriage?" When students turn to the back of the book to check their answer, it simply says, "no," absent any sort of justification, reasoning or argument. What would that tell us about reading and literature? And yet mathematics textbooks do that very thing!

MATH IS SISYPHEAN

Worst of all, the liturgy of a modern math classroom teaches students that the reward for successfully completing bad "mathematical" tasks is worse mathematical tasks. Once you've finished tonight's homework, there will be tomorrow's—equally meaningless, but harder! Like Sisyphus, students roll the boulder of their homework up the hill every night only to have it roll down the next day during class.

Good mathematicians can tackle upwards of thirty sort-of-related problems per hour! Good mathematicians spend as much time on scientific notation as they spend on the idea of an infinitude of primes! Good mathematicians spend as much time on graphing quadratic inequalities as they spend on the fundamental theorem of arithmetic! Good mathematicians realize that all topics are equally important and worth the same time and energy!

No one could mistake this sort of work for the good life.

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Of course, it doesn't have to be like that, and hasn't always been like that. Classical educators are uniquely positioned to teach math in a profoundly human and humane way because we recognize we are part of a long tradition aimed at human flourishing.

If I had to boil it down to one thought, it'd be this: when we talk humanities pedagogy, we should stop saying "except in math." When we talk about good pedagogy in, say, literature or history or rhetoric, we're also talking about good pedagogy in math. There's a profound unity to our intellectual life, and if we recognize that, it transforms how we think about teaching mathematics.

Teaching the humanities means putting the students face-to-face with something powerful and outside themselves: the great texts of our shared intellectual heritage. Those texts then work their way into the hearts of our students and form them into the sorts of people they are meant to be. It's the same in math. Mathematical texts are often good problems, and sometimes they good solutions, too. To be good, a problem must ride a razor's edge: it can't be so trivial that a young mathematician will see the answer in five minutes, and it can't be so hard that it takes a whole class six weeks to work through. Ideally, a good mathematical text should be, to borrow a word from Andrew Smith, dean of academics at Veritas Academy, "apocalyptic," that is, it should reveal some broader pattern or truth.

Once we've found a good problem or text, we want the students to talk about it, just like they would in a literature or history class. This is usually wildly countercultural for them, so much of my job as a math teacher is subverting and replacing their cultural expectations about mathematics.

Students typically walk into our classes believing that authority is the source of mathematical truth—the authority in the back of the book. It's perplexing to them not to have that authority available. Students typically believe that the teacher's role in class is to explain and the students to be explained at. It's challenging to them to bear much of the explanatory burden themselves. Students typically believe that math is a deeply individualistic pursuit. This is trickier to unpack. The refrain I use with my students is that the goal of mathematical discussion is to first convince yourself that you're correct, then convince your peers, and then convince me. So our discussions revolve around reasoning and persuasion, also deeply countercultural in mathematics. The discussions aim at knowing transcendent truths together in community.

Forming that mathematical community is a process deeply rooted in the formation of Christian character. The central image I use when talking to my students about how to form their mathematical community is the body of Christ. As St. Paul writes in his letter to the Corinthians:

[T]here are varieties of gifts, but the same Spirit; and there are varieties of service, but the same Lord; and there are varieties of activities, but it is the same God who empowers them all in everyone. To each is given the manifestation of the Spirit for the common good.

Some students conjecture, some test conjectures, some write programs, some draw diagrams, some point out errors, some work particular examples. Each contributes meaningfully in the search for truth. How different a picture from a modern math classroom!

So, what do mathematicians make? What is the proper telos of mathematical work? Trusting the textbooks we were raised with, we might think some sort of contextfree numerical answer. But according to G.H. Hardy, mathematicians are artists whose medium is "patterns in ideas." I find that compelling. How do mathematicians communicate their art? They write to one another. Much of mathematics has been transacted over letters and papers. We should join the rest of our sibling humanities and do the same. Once we have settled on a path through a problem, it's time for each student to write his or her own paper synthesizing and explaining that path as elegantly and clearly as possible.

This terrifies the students the first time it happens math has usually been the lone respite from paper writing! So we do the first paper together in class, writing it sentence by sentence. We talk about each sentence, whether it can be whittled down just a little bit more, whether it conveys precisely what we want it to convey. We gradually weave those sentences together into a cogent explanation that would be convincing to someone approaching the problem for the first time.

For a discipline that is notionally about eternal and perfect truths, high school mathematics can leave a lot on the table. In book seven of *The Republic*, Plato notes that:

... for [practical] purpose[s] a very little of either geometry or calculation will be enough; the question relates rather to the greater and more advanced part of geometry – whether that tends in any degree to make more easy the vision of the idea of good; and thither, as I was saying, all things tend which compel the soul to turn her gaze towards that place, where is the full perfection of being, which she ought, by all means, to behold.

If we teach mathematics like one of her peers, the humanities, we can incline our students, ever so gently, towards the vision of the good. If we teach mathematics like the vocational, authoritarian slog that so many of us experienced, we can ensure that most students regard math as a horror to be endured, like their parents endured before them.

Mathematics, more than any other subject, forces a school to decide what it's really about. At Ad Fontes, our goal isn't just college acceptance; it's human flourishing. Our goal isn't crassly maximizing our students' future earning; it's ensuring that they see the image of God in themselves. It's not building a materially richer world; it's building a world more like the kingdom of heaven. Teaching mathematics in harmony with her sister humanities is an essential part of that project.