The Rise and Fall of Reason

by Dr. Mitchell Stokes

Genuine mathematical understanding is like a three-legged stool. Doing calculations or deriving theorems is only one of the legs. The other two legs are math's history and philosophy, respectively. I've tried sitting on a one-legged stool, and it's hard. I spent the better part of twenty years learning the grammar of mathematics–its recipes and techniques. I was good at it, too. But I felt cheated when I discovered that there was more to mathematics–so *very* much more. For example, did you know that a *mathematician* began Western civilization's millennia-long search for intellectual certainty, a search that has led to various forms of idolatry? Thales of Miletus (ca. 600 BC) was, in fact, the West's *first* mathematician. He was also its first philosopher. And its first scientist. He initiated our epistemological search by refusing to invoke the Homeric gods as the cause of natural phenomena; rather, he sought *rational* explanations for the cosmic order. Nature, he believed, doesn't behave according to the whims of erratic divine beings. On the contrary, nature is ultimately reasonable and, furthermore, humans are capable of discerning its rational structure. He passed on this belief to his pupil Pythagoras, of Pythagorean theorem fame. Pythagoras, going a step further than Thales, concluded that nature's structure is not merely rational but *mathematical*. A century or so later, Plato–himself a Pythagorean–then set the West's scientific and metaphysical agenda: describe the cosmos in mathematical terms.

Plato's pupil, Aristotle, proposed a method for meeting this challenge. In fact, it was a method by which *all* subjects could be systematically developed and organized. Or so Aristotle supposed. According to his method, each subject or "science"–whether it was mathematics, mechanics, or metaphysics–would begin with fundamental and indubitable assumptions (the *axioms*). These assumptions, in other words, must be absolutely certain. "Well begun is half done," Aristotle said in his *Politics*. From these unquestionable foundations, we then reason to further truths (the *theorems*), thereby building the rest of that particular science. Only if we're confident in our axioms can we be confident in what we derive from them–and then only if we can trust our reasoning. So Aristotle invented the discipline of logic to help with this.

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Although Aristotle intended that his *axiomatic method* be used for any subject, he had modeled it on mathematics. This is because, for the Greeks, mathematics was already the standard for intellectual certainty. It still is today. For most of us.

The famous mathematician Euclid trained at Plato's Academy and so was steeped in Pythagorean ideas. He also, quite naturally, used Aristotle's axiomatic method for his *Elements* (ca. 300 BC). The *Elements* is a compilation of classical Greek mathematics and contains what we now, for obvious reasons, call "Euclidean geometry," the geometry we learned in high school. Euclid could not have possibly foreseen its influence; it became the West's intellectual archetype for the next two thousand years. And so the axiomatic method–a mathematical method–became the West's only foolproof way to certainty in any subject.

The method's promise of assurance enticed thinkers like Descartes, Hobbes, Spinoza, Bacon, Galileo, and Newton to axiomatize their own, non-mathematical theories. With it, Newton, for example, achieved at last what the Greeks had set out to do centuries earlier, namely, to describe the rational structure of the universe with mathematics. His *Principia Mathematica* (ca. 1700) was the culmination of the scientific revolution. In the *Principia*, Newton mathematized the movements of heavenly and earthly phenomena. By assuming his celebrated three laws of motion, he derived, among other things, his law of universal gravitation. If that weren't enough, he invented calculus to help him, further supporting the view that mathematics was the ultimate path to truth.

It would be difficult to overstate the effect that Newton's achievements had on Europe's intellectual temperament. The resulting optimism in man's rational powers bordered on profligate. Odes and poems were written in Newton's honor. With mathematics–a purely *mental science*–Newton had at last revealed the secret workings of the *physical* cosmos. The mathematization of motion was the main technical

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achievement of the scientific revolution. But more importantly, the revolution unseated traditional cultural authorities. Although the influence of the Church and the Ptolemaic system had been gradually diminishing since the Middle Ages, it was Newton's *Principia* that officially ended their rule. And so, by inaugurating reason as the final arbiter of truth, the *Principia* ushered in the Enlightenment. In fact, Immanuel Kant, the Enlightenment's unofficial spokesman (and perhaps second only to Plato in overall influence), found his primary inspiration in the successes of Newtonian mechanics. According to Kant, the Enlightenment's motto was "Have courage to use your own understanding!" The modernist spirit had come of age. But one authority survived: Euclid's *Elements*, for it was the very incarnation of pure reason.

During the 1800s, however, and roughly a hundred and fifty years after Newton's triumph, mathematicians discovered a problem with the *Elements*. Despite the fact that Euclid had begun with axioms so obvious that denying any one of them would be absurd, mathematicians found that they could replace one of these axioms with its negation (while keeping the other axioms) and still derive a perfectly consistent geometrical system. In fact, they discovered *two* such systems. These were alternative geometrical worlds in which the sum of the interior angles of a triangle *isn't* 180 degrees and "straight lines"–still the shortest distance between two points–can curve back on themselves! It's hard for us to identify with the resulting shock but bear in mind that an alternative to Euclidean geometry would have been considered as possible as a square circle.

The one consolation, though, was that ordinary Euclidean geometry described the real world. To put it differently, at least Euclidean geometry was *true*. The "non-Euclidean" geometries could still be seen–at first–as merely mathematical games, albeit disturbing ones. But in the early 1900s a new theory of gravity–Einstein's general theory of relativity–employed one of the new geometries to describe real-life physical space. Therefore, if general relativity is true, Euclidean geometry is strictly speaking *false*.

But how could this be? The *Elements* had been the paradigm of truth and certainty for over 2000 years. It's credentials were impeccable. It had been the exemplar for all knowledge. Not only that; this was *mathematics,* the one place we find *absolute* certainty. How could mathematical "truths" be false, especially a truth so obvious that it qualifies as an unquestionable assumption?

Hoping to regain the promise of certainty, mathematicians and philosophers responded to this crisis with a flurry of work (including the invention of *symbolic* logic). But no consensus was ever reached regarding the nature of mathematics.

Many skeptically-minded folks (we might call them postmodernists) were quick to take note of this, becoming overly suspicious of reason: "People have mistakenly believed that there are absolute moral standards, but there aren't even absolute *mathematical* standards. See, we told you there aren't absolute truths." Not the finest bit of reasoning, but you can appreciate the feelings behind it. Imagine you discover that your mom has been systematically lying to you your entire life. If you can't trust your mom, who can you trust? Similarly, who can you trust, if not Euclid?

So then, a second revolution had occurred, one in which Euclid himself had been overthrown. Whereas the scientific revolution resulted in excessive optimism in man's rational faculties, the non-Euclidean revolution sparked an exaggerated sense of pessimism. Both of these common attitudes exist in our culture today, schizophrenically side by side. And both can be traced back to *mathematical* revolutions. But in each case–whether extreme optimism or extreme pessimism–man is taken as the measure, either by way of his own reason or else by his own judgment *on* reason (presumably using *reason*!) Neither of these extremes should be our response, of course. Reason is a God-given tool, and we can therefore count on its general reliability, even while conceding its fallibility. The search for ultimate certainty is ultimately idolatry. Looking for this kind of certainty is simply yearning to be like God.

My real point, however, (made primarily by showing rather than by telling) is that the history and philosophy of mathematics can actually tell the West's sweeping intellectual story. Through mathematics we can see the spirits of the age. If we desire to understand Western culture (and we should), then understanding mathematics can no longer be seen as a charming option. Yet *understanding* mathematics requires more than technical acumen. As important as the grammar of mathematics is, it is only the first step towards our real goal: genuine understanding.