CREATING MATHEMATICIANS

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THE RIGHT DIRECTION

Have you ever hoped that your math students would demonstrate true mathematical thinking during class? My students and I can easily treat math like a list of to-do items or a recipe to follow. When I teach mathematics that way, I have found that students cannot do a problem that is slightly different than the example given to them because I have not taught them to think mathematically! Though I've tried to move away from a "plug-and-play" method of teaching mathematics previously, I am currently trying a brand-new approach. My aim in this article is to give you a window into my current pedagogy and see if you want to join me in developing a curriculum aimed at teaching students to think mathematically.

The approach I am using is based on the pedagogy utilized by Philips Exeter Academy which is a problem-based, Harkness-style curriculum. It is definitely challenging, and I have gone through various stages of "What am I doing? Why am I doing this? Can I please go back to that plain old textbook? It is so much easier and less stressful!" Yet, even in the midst of the challenges, I can already see a noticeable difference in the way these kids are thinking. They are thinking mathematically!

LEARNING FOR THEMSELVES

"Telling is not teaching" has rung in my mind since the first time I read Gregory's *The Seven Laws of Teaching.*[1] The whole point of a Harkness-based approach is that the student is the one doing the thinking. The student's mind is the focus at the beginning, the middle, and the end of the process. Rule six in Gregory's laws states: "Learning is thinking into one's own understanding a new idea or truth." For example, in the past, I would need to make sure that we derived the distance formula as a class since I did not want my students to blindly follow a formula. And then, whenever a student was having difficulty finding the distance between two points, we would graph the points and use the Pythagorean theorem to find the distance. Contrarily, Exeter Mathematics 2 does not provide the distance formula to the student. It trains a progression of learning. First, students discover one derivation of the Pythagorean theorem. Then, students create right triangles on a coordinate graph and use the Pythagorean

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theorem to find distances. They discover that solving for a distance is a necessary stepping stone to solve other kinds of problems, none of which simply ask "find the distance between these two points." These problems do not look like distance formula problems. Students might be asked to find the speed of a particle with a position described by a parametric equation, prove two triangles congruent by SSS, determine how far a triangle was translated by vector [5,4], or decide which point is closer to a given line. Because of this approach, in no time at all, students are telling *me* the distance formula. Additionally, in the process, they are discovering connections between many ideas: slope, the Pythagorean theorem, rates, distance, congruence, perpendicular bisectors, and angle bisectors. These connections cause students to be more creative when solving new problems.

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What is wonderful about a problem-based approach is that no prescribed method is given to solve each problem. Instead, students have to learn to ask the right kinds of questions. They learn that problems can be solved multiple ways, and they begin to ask themselves, "Is there a more effective way to approach this?" before taking a particular path. As an example, last year my geometry class tackled this review question:

> *You are one mile from the railroad station, and your train is due to leave in ten minutes. You have been walking at a steady rate of 3 mph, and you can run at 8 mph if you have to. For how many more minutes can you continue walking, until it becomes necessary for you to run the rest of the way to the station? (Exeter Mathematics 2, 2017, #196).*

One student solved the problem by trial-and-error and another created an equation. An eighth grader decided to graph. He said, "I started graphing a line from the origin with the person's walking rate as the slope. Then, I graphed another line starting at the point (10 min, 1 mile) and worked backwards using the running rate as the slope. Where these two lines intersect is the time and place at which the person must start running." There are benefits to talking through all three solutions in class. One is that the students are given a review of the three methods. Another is the exposure to the connection between the three solutions. For instance, speed is seen in the terms of the equation, in the slopes of the line, and within the trial-and-error data. Additionally, students often realized there might be a more effective method which, in turn, prompted them to ask that question of themselves more often. When students must decide the method to solve a problem, they are becoming mathematicians.

CONVERSING AROUND THE TABLE

Gregory's Law of the Language says, "Not what the speaker expresses from his own mind, but what the hearer understands and reproduces in *his* mind, measures the exact communicating power of the language used" (70). "Secure from him as full a statement as possible of his knowledge of the subject, to learn both his ideas and his mode of expressing them, and to help him to correct his language" (77). "That teacher is succeeding best whose pupils talk most freely upon the lessons" (78).

I have learned over the years that there are ways of teaching math that stifle purposeful questions and dialogue. Students generally want to know if they got the answer right or wrong. Exeter math counters that tendency. Having students present their work to other students on a daily basis reveals the gaps in their thinking. We can think students understand us since they are able to "do" math, but we are often mistaken. An example from my seventh-grade algebra class early this year reminds me of this fact. The seventh-grade algebra class spent forty minutes discussing an answer to this problem while

I watched:

> *Tory goes shopping and buys pencils and notebooks. If Tory buys a total of 8 items, p of which are pencils, write an expression for the number of notebooks Tory buys" (Exeter Mathematics 1, 2018, #34).*

My only input was relational such as, "Sarah, when someone makes a comment to you, it is important for you to respond. A thank you is sufficient." If my ten-year -younger self were watching this class, I would think I was a little crazy. This problem is simple. There are ways to teach this concept such that students understand and are able to apply their understanding but in a much faster way. We could get through so much more material! As I observed their dialogue, however, I realized how much misunderstanding abounded. Students had the meaning of a variable and the meaning of a label mixed up. Many had a hard time realizing that 8-p represented a number. In the end, their discussion brought the entire class to a complete understanding not just of this particular problem, but also the meaning of a variable, the best way to use labels, and appropriate mathematical notation. I also got a glimpse into their insightful questions and ideas. Forty minutes felt much too long to spend on this. However, it is saving five minutes here and there which will add up to much more than forty minutes over the rest of the year. Classroom discussion also gives the much needed opportunity to teach our students how to interact with each other truthfully and graciously. Learning how to disagree, defend an opinion, and ask difficult questions are very important skills. This cannot be taught by lecture; it must be taught by example and practice.

COURAGE AND HOPE

As I speak with a logic student who is exhibiting the traits I am describing, I wonder to myself what would happen if we could continue this kind curriculum all the way until her senior year? What if we could rewrite our curriculum so students created a proper biblical view of mathematics along the way? What if the students became like Kepler who gave great praise and thanks to God when he discovered that the planets moved along the path of an ellipse? Though this curriculum may not be the exact way to go, I believe that it is pointing in the right direction. In these last few years, I have been given hope that we can create an excellent classical Christian math curriculum. I see glimpses of what is possible. I also see that there is a big mountain to climb between where we are now and where we could be without clearly seeing the path. God provides and directs. There IS a better curriculum for classical Christian schools waiting to be written. Will you join me in forging this path?

ENDNOTES:

1. John Milton Gregory, *The Seven Laws of Teaching,* Veritas Press, 2004.